



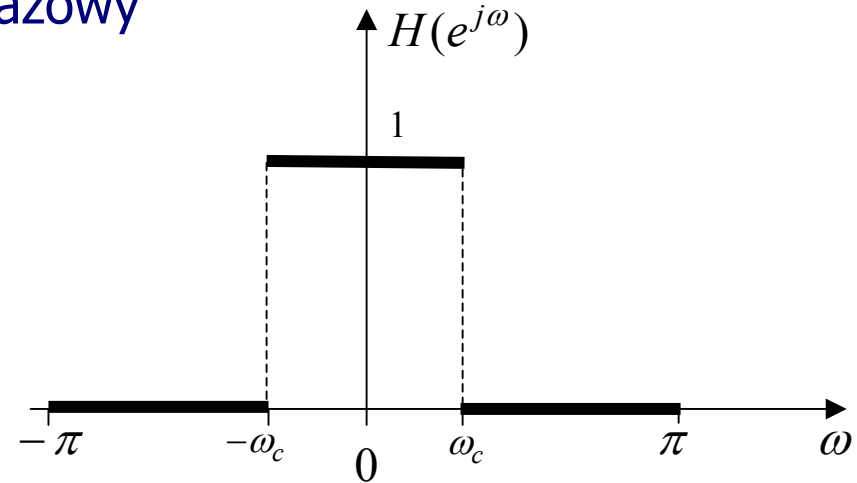
# Filtry idealne

Do filtrów idealnych selektywnych zaliczamy m.in. filtr dolno-przepustowy, filtr górno-przepustowy, filtr pasmowo-przepustowy i filtr pasmowo-zaporowy.

## Filtr idealny dolno-przepustowy zero-fazowy

$$H(e^{j\omega}) \triangleq \begin{cases} 1, & 0 < |\omega| < \omega_c \\ 0, & \omega_c < |\omega| < \pi \end{cases}$$

$$\omega \in (-\pi, \pi), \quad f = \frac{\omega}{2\pi}, \quad f \in (-1/2, 1/2)$$



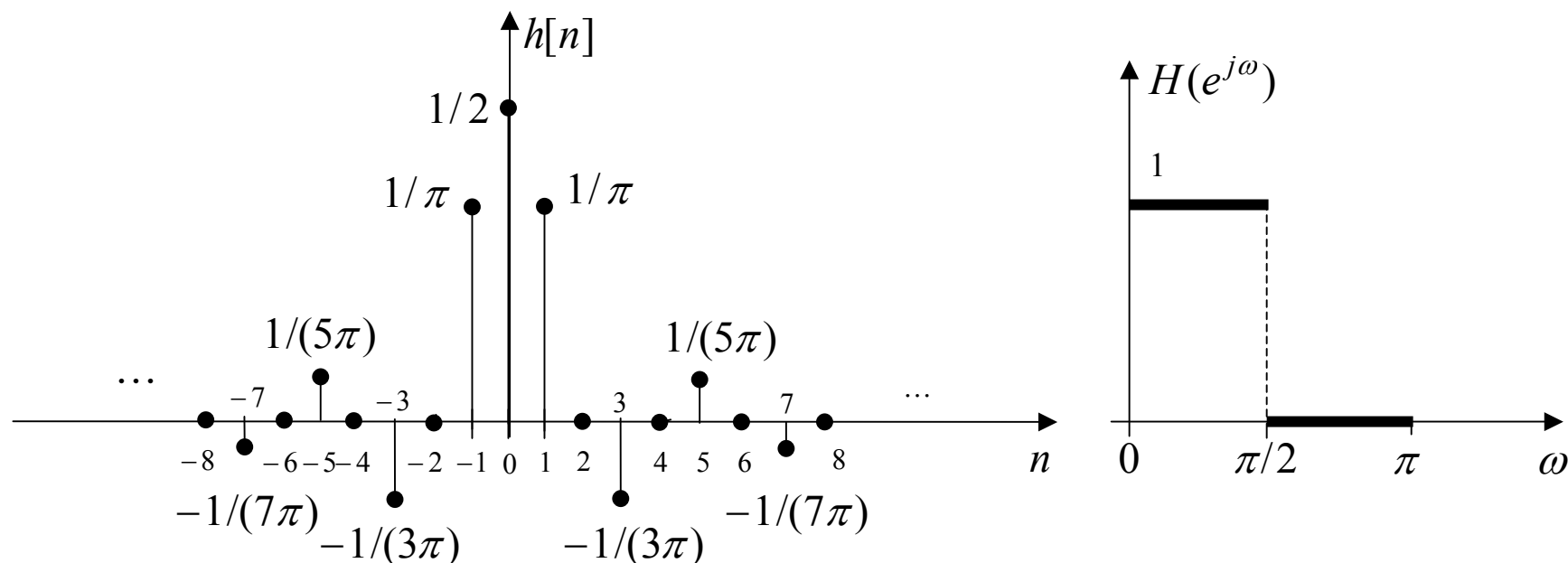
$$\begin{aligned} h[n] &= \text{IDTFT}\{H(e^{j\omega})\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega n} d\omega = \\ &= \frac{1}{2\pi} \left[ \frac{1}{jn} (e^{j\omega_c n} - e^{-j\omega_c n}) \right] = \frac{1}{\pi n} \sin(\omega_c n) = \begin{cases} 2f_c \frac{\sin(\omega_c n)}{\omega_c n} = 2f_c \text{sinc}(\omega_c n / \pi), & -\infty < n < \infty, \quad n \neq 0 \\ 2f_c, & n = 0 \end{cases} \end{aligned}$$

$$\text{sinc}(0) = \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

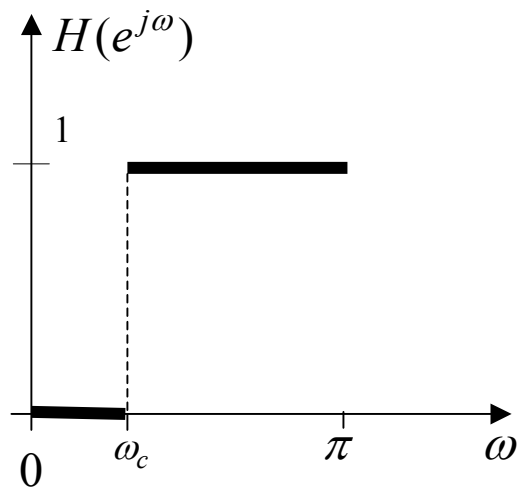
## Przykład. Filtr idealny dolno-przepustowy pół-pasmowy zero-fazowy

Dla  $\omega_c = \frac{\pi}{2} \rightarrow f_c = \frac{1}{4}$  mamy  $h[n] = h_{\text{HBF}}[n]$  i

$$h_{\text{HBF}}[n] = \frac{1}{\pi n} \sin\left(\frac{\pi}{2} n\right) = \begin{cases} \frac{1}{2} \frac{\sin\left(\frac{\pi}{2} n\right)}{\frac{\pi}{2} n} = \frac{1}{2} \text{sinc}\left(\frac{n}{2}\right), & -\infty < n < \infty, \quad n \neq 0 \\ \frac{1}{2}, & n = 0 \end{cases}$$



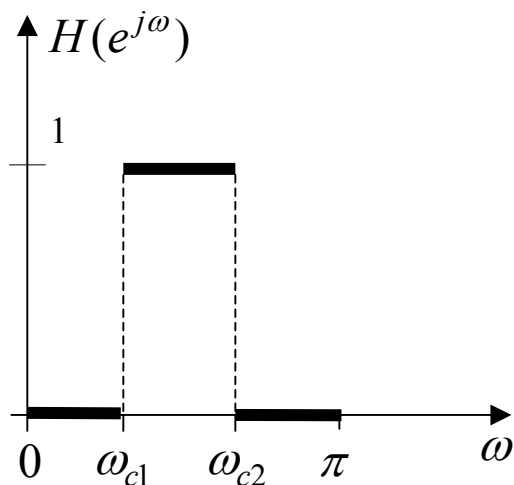
## Filtr idealny górno-przepustowy zero-fazowy



$$H(e^{j\omega}) \triangleq \begin{cases} 1, & \omega_c < |\omega| < \pi \\ 0, & 0 < |\omega| < \omega_c \end{cases}$$

$$h[n] = \begin{cases} -2f_c \frac{\sin(\omega_c n)}{\omega_c n} = -2f_c \text{sinc}(\omega_c n / \pi), & -\infty < n < \infty, \quad n \neq 0 \\ 1 - 2f_c, & n = 0 \end{cases}$$

## Filtr idealny pasmowo-przepustowy zero-fazowy

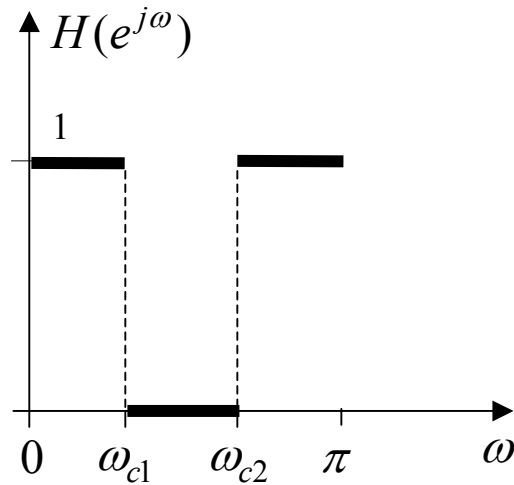


$$H(e^{j\omega}) \triangleq \begin{cases} 1, & \omega_{c1} < |\omega| < \omega_{c2} \\ 0, & 0 < |\omega| < \omega_{c1} \text{ i } \omega_{c2} < |\omega| < \pi \end{cases}$$

$$\omega_{c2} > \omega_{c1}$$

$$h[n] = \begin{cases} 2f_{c2} \frac{\sin(\omega_{c2} n)}{\omega_{c2} n} - 2f_{c1} \frac{\sin(\omega_{c1} n)}{\omega_{c1} n}, & -\infty < n < \infty, \quad n \neq 0 \\ 2(f_{c2} - f_{c1}), & n = 0 \end{cases}$$

## Filtr idealny pasmowo-zaporowy zero-fazowy



$$\omega_{c2} > \omega_{c1}$$

$$H(e^{j\omega}) \triangleq \begin{cases} 1, & 0 < |\omega| < \omega_{c1} \text{ i } \omega_{c2} < |\omega| < \pi \\ 0, & \omega_{c1} < |\omega| < \omega_{c2} \end{cases}$$

$$h[n] = \begin{cases} 2f_{c1} \frac{\sin(\omega_{c1}n)}{\omega_{c1}n} - 2f_{c2} \frac{\sin(\omega_{c2}n)}{\omega_{c2}n}, & -\infty < n < \infty, \quad n \neq 0 \\ 1 - 2(f_{c2} - f_{c1}), & n = 0 \end{cases}$$